MEERUT INSTITUTE OF ENGINEERING AND TECHNOLOGY

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Pre University Test (PUT) : Odd Semester 2022-23

Course/Branch	: B Tech – CSE/ECE/ME	Semester	: III
Subject Name	: Engg. Mathematics IV	Max. Marks	: 100
Subject Code	: KAS 302	Time	: 180 min

CO-1 : On completion of this course, the student will be able to apply PDEs to solve mathematical problems.

CO-2 : On completion of this course, the student will be able to apply the concepts of PDEs in engineering problems.

CO-3 : On completion of this course, the student will be able to apply the concepts of descriptive Statistics.

CO-4 : On completion of this course, the student will be able to learn probability and apply it through various discrete and continuous distributions in real life.

CO-5 : On completion of this course, the student will be able to apply statistical quality control in manufacturing process.

Section – A # 20 Marks

Attempt **ALL** the questions.

(10 x 2=20 Marks)

Q. No.		COx	Question	
1	(a)	CO1	Solve the partial differential equation $\sqrt{p} + \sqrt{q} = 1$.	[K3]
	(b)	CO1	Calculate particular integral (P.I.) of $(D - 3D' + 2)z = e^{x+2y}$.	[K3]
	(c)	CO2	Write down the two-dimensional heat equation.	[K2]
	(d)	CO2	Explain the telegraph equations.	[K2]
	(e)	CO3	What is meant by skewness?	[K2]
	(f)	CO3	Write the normal equations of multiple linear regression of Y on Z and X.	[K2]
	(g)	CO4	State the Baye's theorem.	[K2]
	(h)	CO4	If the probability density function $f(x) = \begin{cases} kx^3, & if\ 0 \le x \le 3 \\ 0, & elsewhere \end{cases}$, find the value of k.	[K3]
	(i)	CO5	When is the test statistic $F = \frac{s_1^2}{s_2^2}$ is used?	[K2]
	(j)	CO5	Explain One-way classification in ANOVA.	[K2]

Section – B # 30 Marks

Attempt **ALL** the questions.

(5 x 6=30 Marks)

Q.2 (CO-1): Use Cauchy's method of Characteristics to solve the following first order partial differential equation

$$u_x + u_y = 1 + \cos y, \qquad u(0, y) = \sin y. \qquad \text{[K3]}$$

OR

Solve the following partial differential equation by using Charpit's method:

$$z^2 = pqxy \text{ where } p = \frac{\partial z}{\partial x}, \ q = \frac{\partial z}{\partial y}. \qquad \text{[K3]}$$

Q.3 (CO-2): A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in position given by $u = \sin\left(\frac{2\pi x}{L}\right)$. If it is release from rest from the position, find the displacement $u(x, t)$. [K3]

<div align="center">OR</div>

An insulated rod of length L has its ends P and Q maintained at $0^\circ C$ and $100^\circ C$ respectively until steady state condition prevails. If end Q is suddenly reduced to $0^\circ C$ and maintained at $0^\circ C$, find the temperature at a distance x from P at time t. [K3]

Q.4 (CO-3): Find the moment generating function (m.g.f.) of the discrete binomial distribution given by

$$P(x) = {}^nC_x p^x q^{(n-x)} \quad (\text{where } q = 1-p)$$

Also find the first and second moments about the mean. [K3]

<div align="center">OR</div>

The pressure of the gas corresponding to various volumes V is measured, given by the following data:

V (cm³)	50	60	70	90	100
P (kg cm⁻²)	64.7	51.3	40.5	25.9	78

Fit the data to the equation $PV^Y = C$. [K3]

Q.5 (CO-4): Show that Poisson distribution is a particular limiting form of the Binomial distribution when p or q is very small, and n is large enough. [K3]

<div align="center">OR</div>

In a bombing action, there is 50% chance that any bomb will strike the target. Two direct hits are needed to destroy the target completely. How many bombs are required to be dropped to give a 99% chance or better of completely destroying the target? [K3]

Q.6 (CO-5): The lifetime of electric bulbs for a random sample of 10 from a large consignment gave the following data:

Item	1	2	3	4	5	6	7	8	9	10
Life in '000 hrs.	4.2	4.6	3.9	4.1	5.2	3.8	3.9	4.3	4.4	5.6

Can we accept the hypothesis that the average lifetime of bulb is 4000 hrs.?
[Given $t_{0.05,9} = 2.26$] [K3]

<div align="center">OR</div>

Following is the data of defectives of 10 samples of size 100 each.

Samples	1	2	3	4	5	6	7	8	9	10
No. of defectives	15	11	9	6	5	4	3	2	7	1

Construct p- chart and state whether the process is in statistical control. [K3]

<div align="center">Section – C # 50 Marks</div>

Attempt ALL the questions. (5 x 10=50 Marks)

Q.7 (CO-1): Attempt any ONE question.

  (a). Solve $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y} - x^2 y$ where $D \equiv \dfrac{\partial}{\partial x}$, $D' \equiv \dfrac{\partial}{\partial y}$. [K3]

  (b). Solve $(x^2 D^2 - y^2 D'^2 + xD - yD')z = \ln x$ where $D \equiv \dfrac{\partial}{\partial x}$, $D' \equiv \dfrac{\partial}{\partial y}$. [K3]

Q.8 (CO-2): Attempt any ONE question.

  (a). A thin metal rod of length L with both ends insulated (so that there is no passage of heat through the ends) and with initial temperature $3\sin\dfrac{\pi x}{L}$ in the rod, Find the temperature function $u(x, t)$. [K3]

  (b). Solve the Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy- plane with $u(x, 0) = 0$, $u(x, b) = 0$, $u(0, y) = 0$ and $u(a, y) = g(y)$ parallel to y-axis. [K3]

Q.9 (CO-3): Attempt any ONE question.

(a) In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are legible:

Variance of $x = 9$,

Regression equations: $4x - 5y = -33$, $20x - 9y = 107$.

What were

- the mean values of x and y
- the standard deviation of y and the coefficient of correlation between x and y ?

[K3]

(b). Calculate the coefficient of correlation between the marks obtained by 8 students in Mathematics and Statistics.

Students	A	B	C	D	E	F	G	H
Mathematics	25	30	32	35	37	40	42	45
Statistics	08	10	15	17	20	23	24	25

[K3]

Q.10 (CO-4): Attempt any ONE question.

(a). Fit a binomial distribution for the following data and compare the theoretical frequencies with the actual ones

x	0	1	2	3	4	5
f	2	14	20	34	22	8

[K3]

(b). In a sample of 1000 cases the mean of a certain test is 14 and S.D is $\frac{5}{2}$. Assuming the distribution to be normal, find

(i) How many students score between 12 and 15?

(ii) How many score above 18?

(iii) How many score below 8?

Given $f(0.8) = 0.2881, f(0.4) = 0.1554, f(1.6) = 0.4452, f(2.4) = 0.4918$.   [K3]

Q.11 (CO-5): Attempt any ONE question.

(a). From the following table regarding the color of eyes of father and son, test if the color of son's eye is associated with that of father.

Eye color of father		Eye color of son	
		Light	Not Light
	Light	471	51
	Not Light	148	230

[Given $\chi^2_{0.05}(1)=3.841$]   [K3]

(b). Distinguish between np-chart and C-chart Following is the data of defectives of 10 samples of size 100 each. Construct np-chart and examine whether the process is in statistical control.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of defectives	6	9	12	5	12	8	8	16	13	7

[K3]